

# STATISTICAL AND COMPUTATIONAL CHALLENGES OF CONSTRAINING GREENHOUSE GAS BUDGETS

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Carnegie Institution for Science

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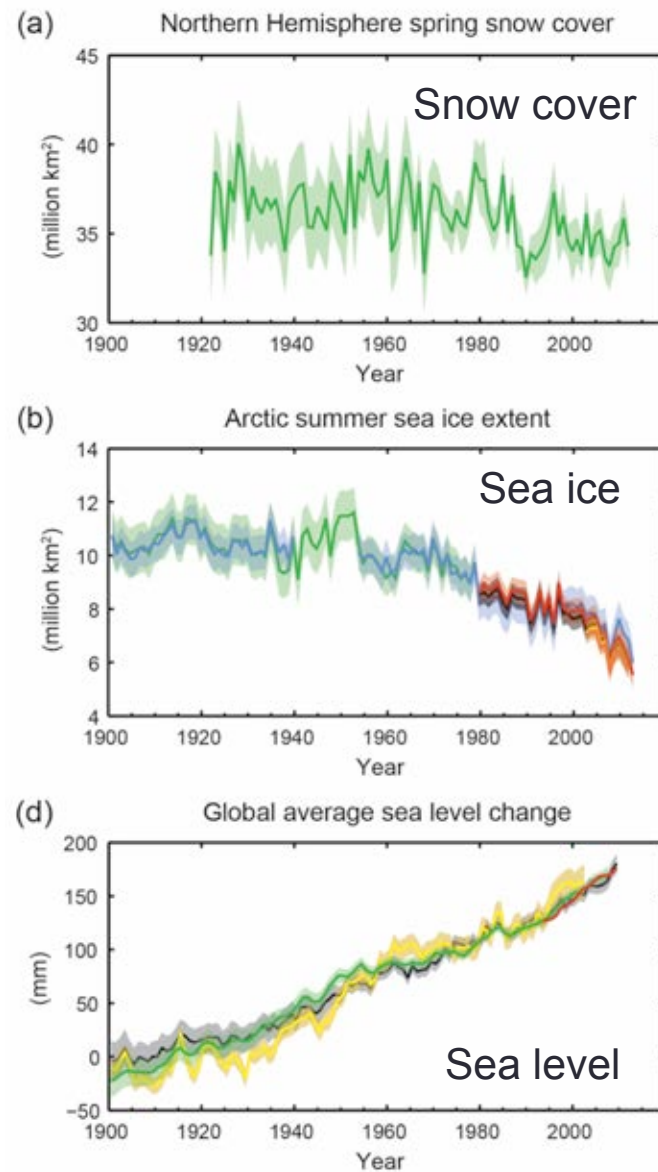
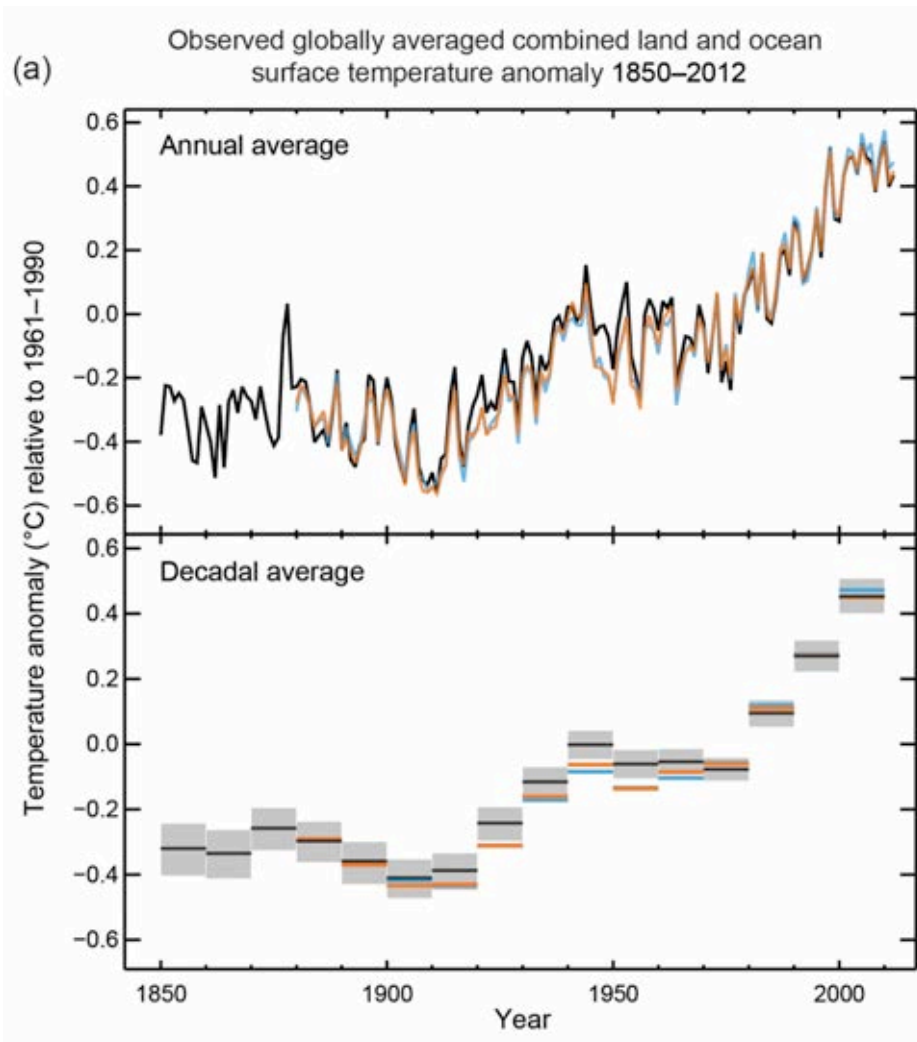
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SCIENCE

DEPARTMENT OF  
GLOBAL ECOLOGY



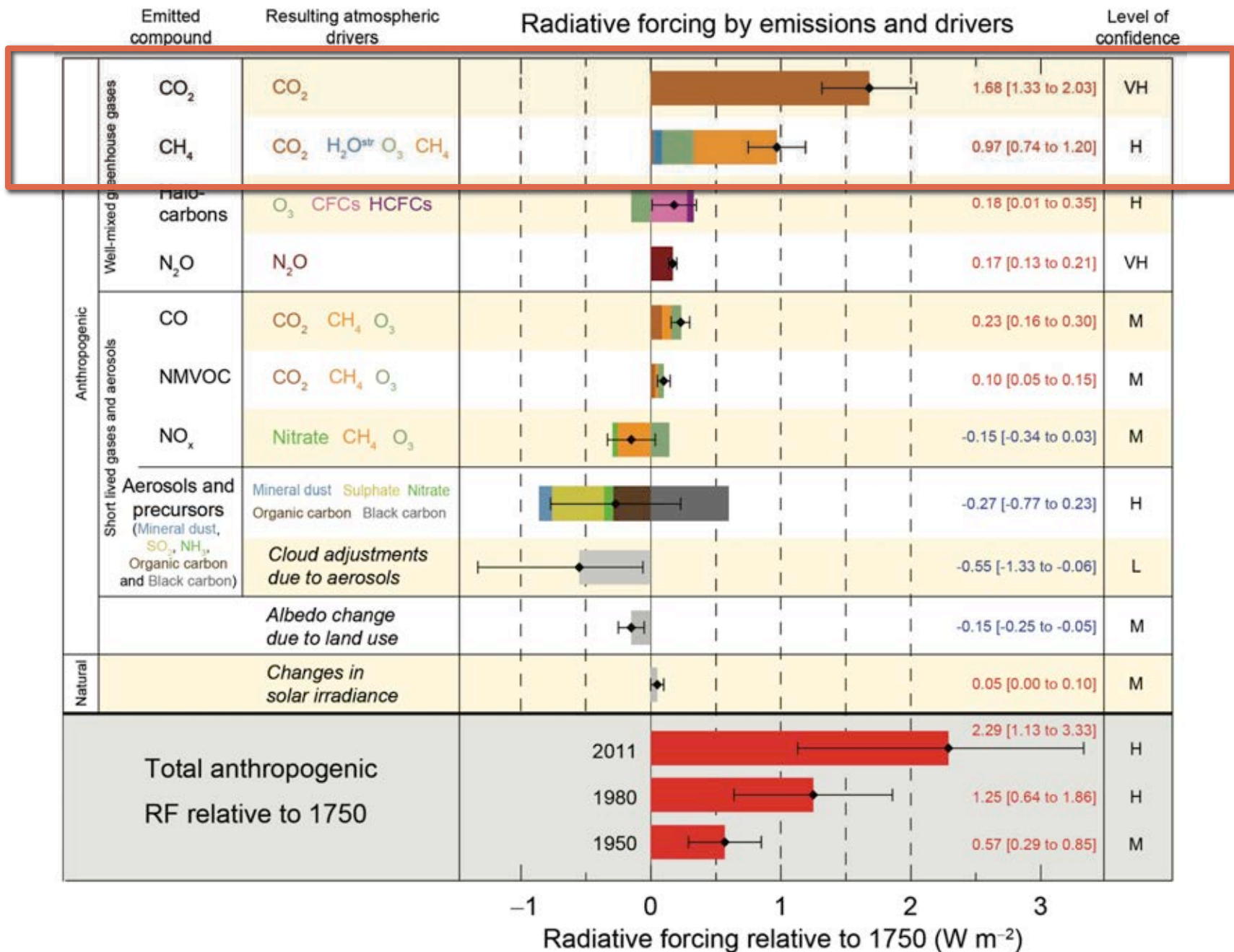
# Take home messages

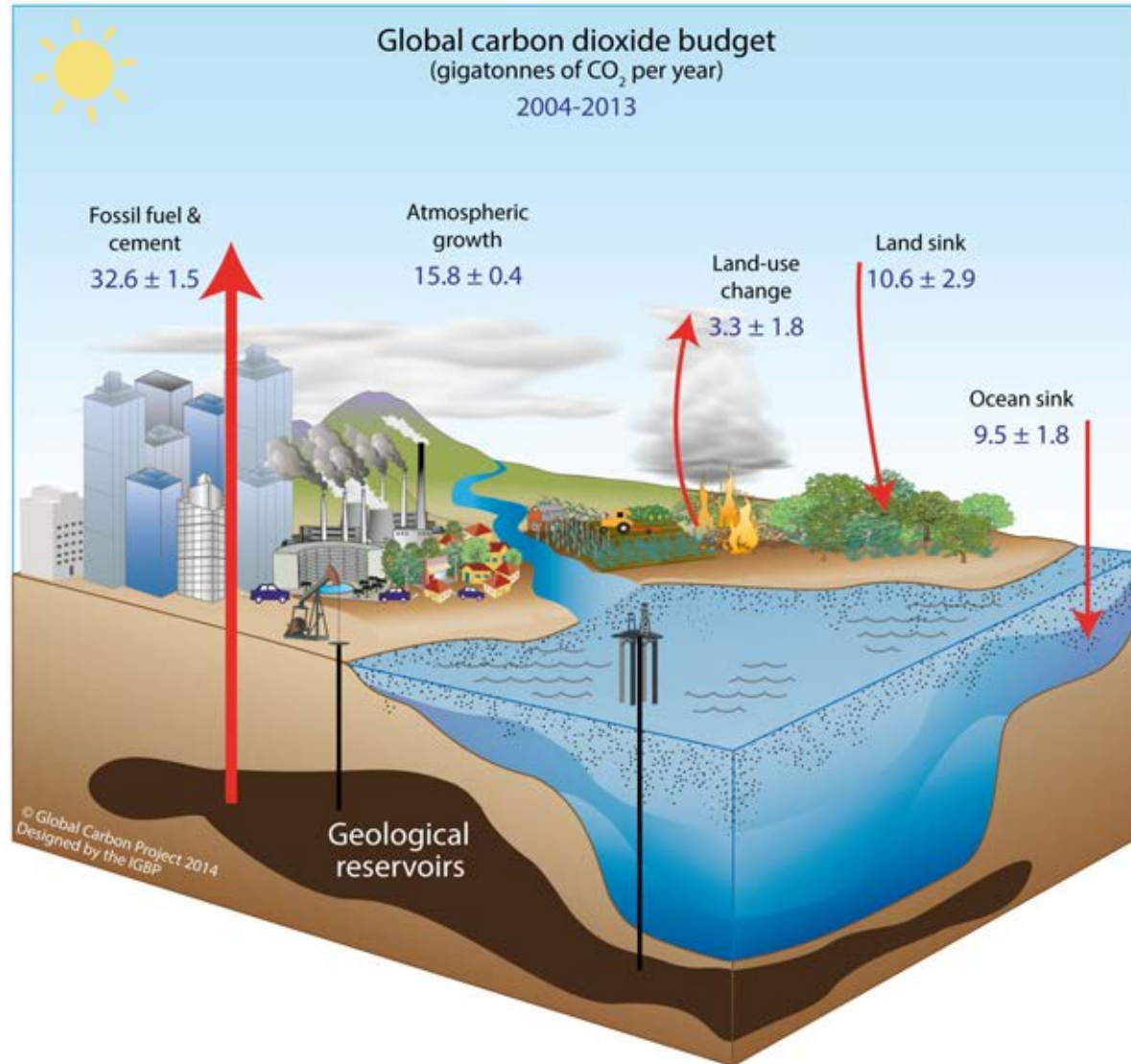
- The need to constrain greenhouse gas budgets inevitably leads to the need for the solution of inverse problems
- These inverse problems:
  - Require (intelligently) choosing among many uncomfortable assumptions
  - Are becoming increasingly statistically sophisticated and computationally demanding
  - Done carefully, can lead to fundamental insights with management and policy implications



# Tioga Pass, January 12 2015

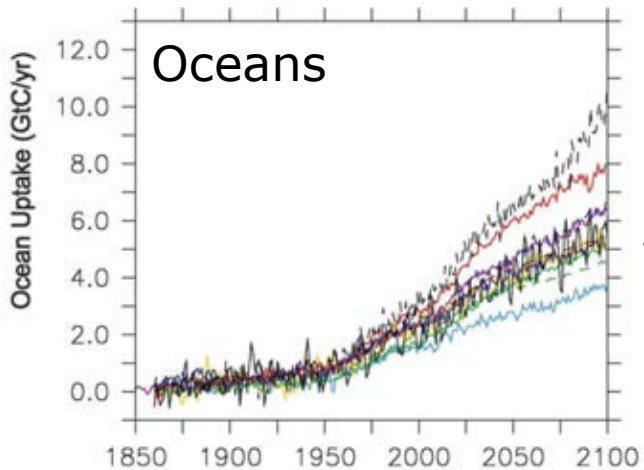
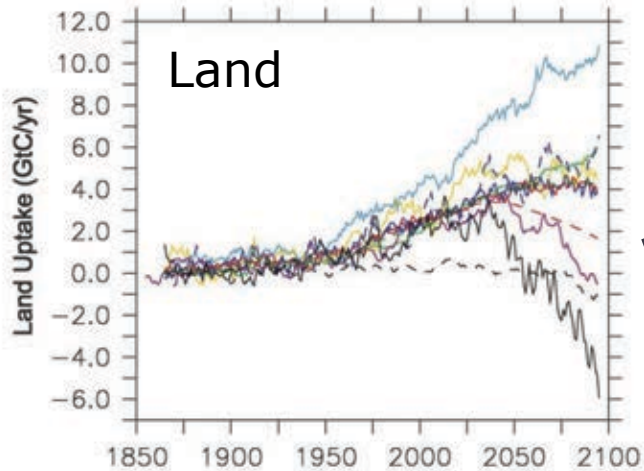




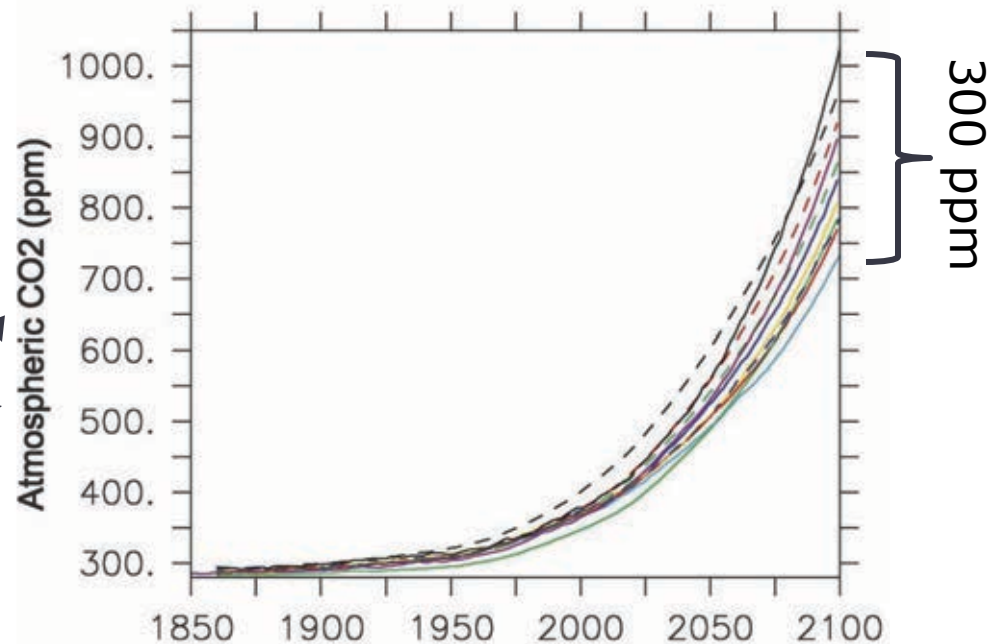


Perturbation of the global carbon cycle caused by anthropogenic activities, averaged globally for the decade 2004–2013 (GtCO<sub>2</sub>/yr)

# The future of natural carbon sinks



Uncertainty associated with the future of natural carbon sinks is one of three major sources of uncertainty in future climate projections



Source: Friedlingstein et al. (2006) showing projections from coupled carbon and climate simulations for several models.

# Paris Agreement

Policy

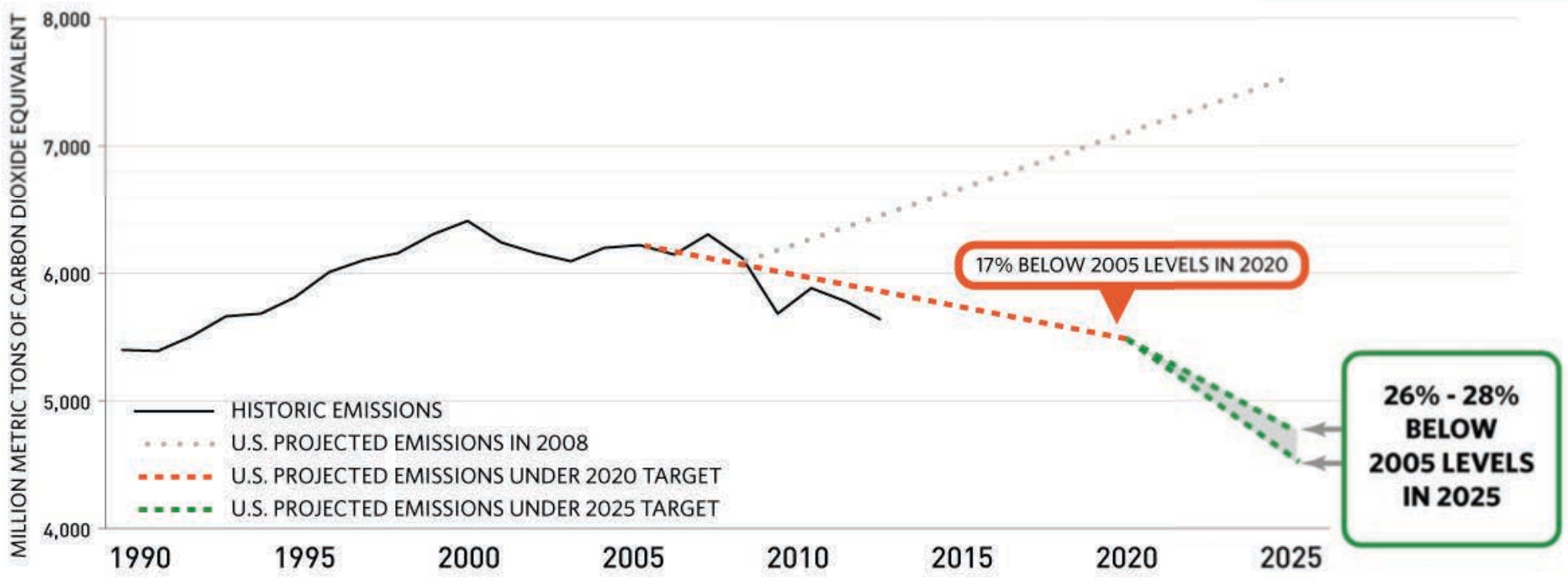
Documentation

At the Paris climate conference (COP21) in December 2015, 195 countries adopted the first-ever universal, legally binding global climate deal.

The agreement sets out a global action plan to put the world on track to avoid dangerous climate



## U.S. EMISSIONS UNDER 2020 AND 2025 TARGETS





# METHANE BUDGET : 2000-09

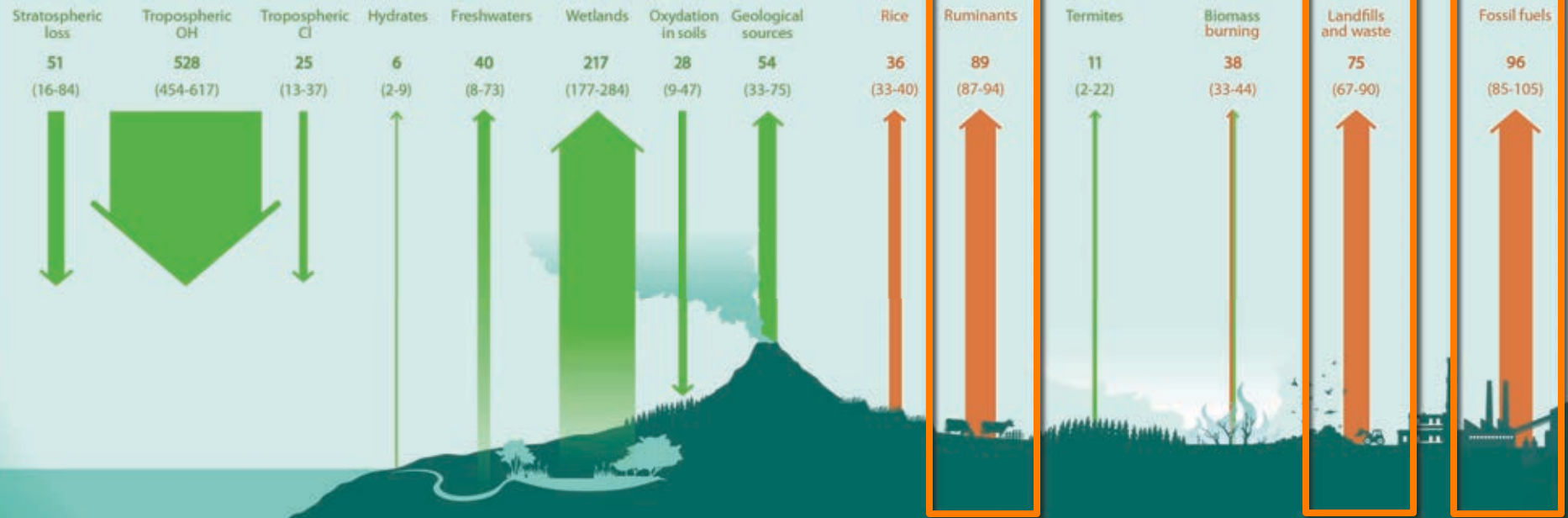
## ATMOSPHERE

Methane reservoir in atmosphere prior to the Industrial Era (in TgCH<sub>4</sub>)

2 007  
± 90

2 960 (+60)

Cumulative changes over the Industrial Era 1750-2009 (decadal growth)



### EXCHANGES BY SOURCE

in teragrams CH<sub>4</sub> / year

→ Natural fluxes

→ Anthropogenic fluxes

→ Combined natural and anthropogenic



# PRIME MINISTER OF CANADA JUSTIN TRUDEAU

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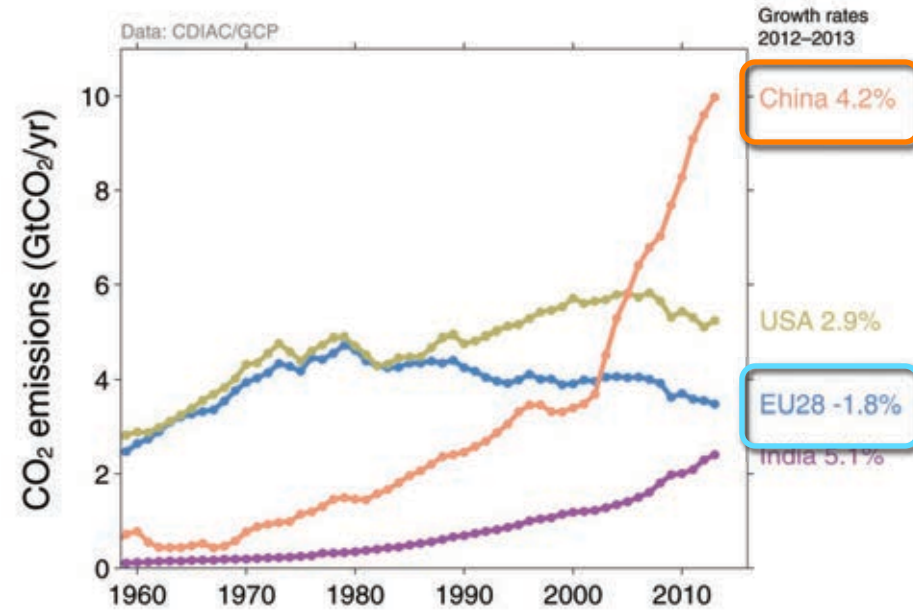
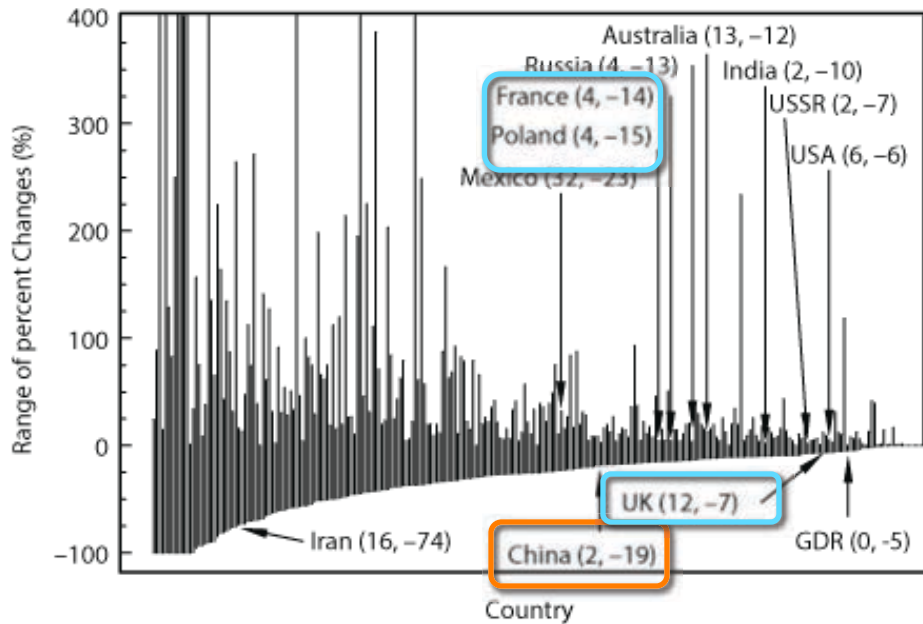
## U.S.-CANADA JOINT STATEMENT ON CLIMATE, ENERGY, AND ARCTIC LEADERSHIP

### **Coordinated domestic climate action**

Building on a history of working together to reduce air emissions, Canada and the U.S., commit to take action to reduce **methane** emissions from the oil and gas sector, the world's largest industrial **methane** source, in support of achieving our respective international climate change commitments. To set us on an ambitious and achievable path, the leaders commit to reduce **methane** emissions by 40-45 percent below 2012 levels by 2025 from the oil and gas sector, and explore new opportunities for additional **methane** reductions. The leaders also invite other countries to join the target or develop their own **methane** reduction goal. To achieve this target, both countries commit to:

# How do we know emissions?

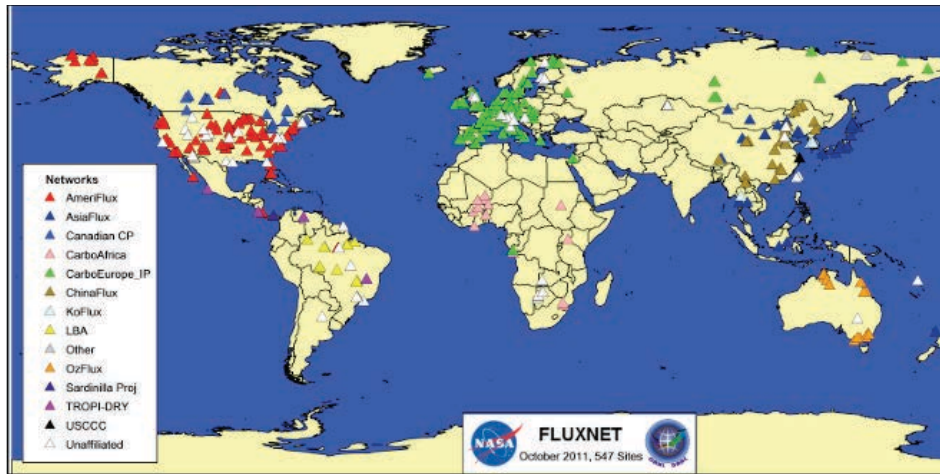
## Self reporting



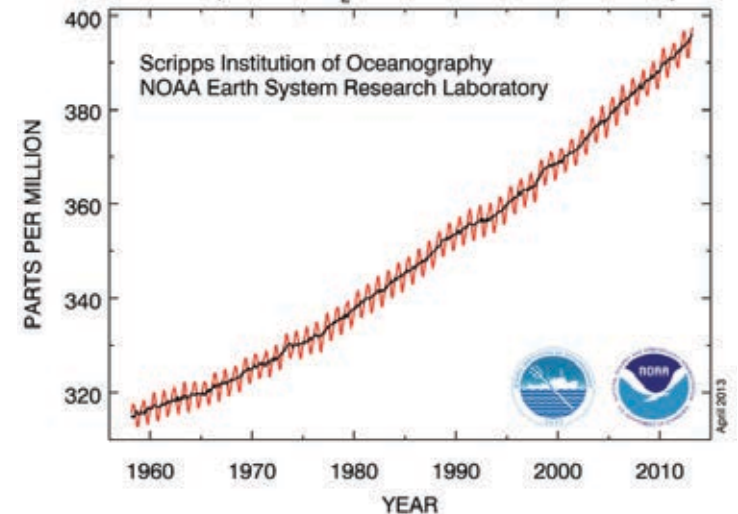
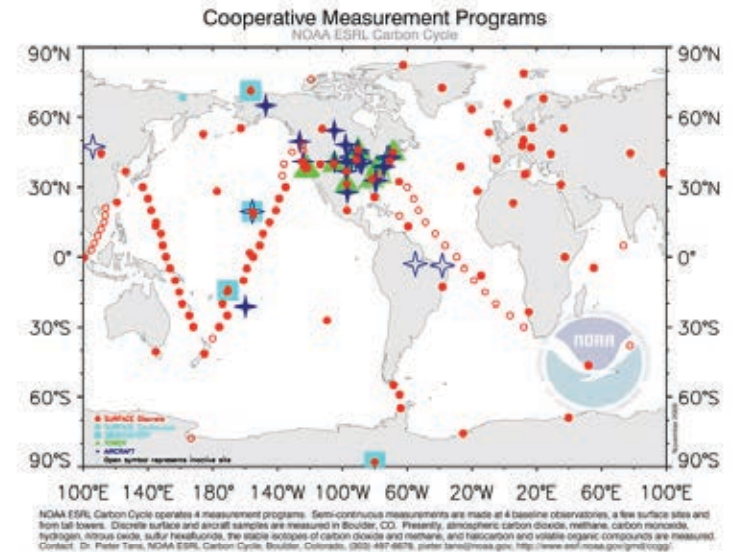
# How do we know emissions? Inventories



# How do we know emissions?

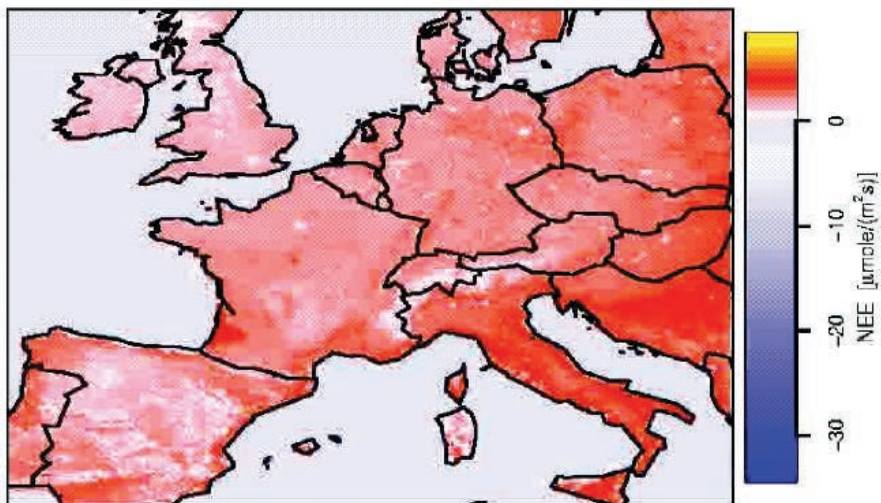


Fluxes (i.e. emissions / uptake)



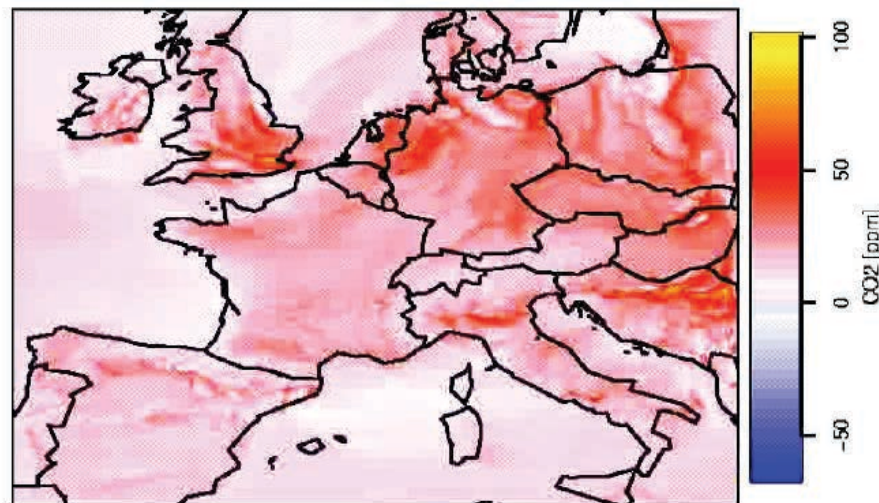
Concentrations

Net Ecosystem Exchange, time 2003-07-02\_01:00:00



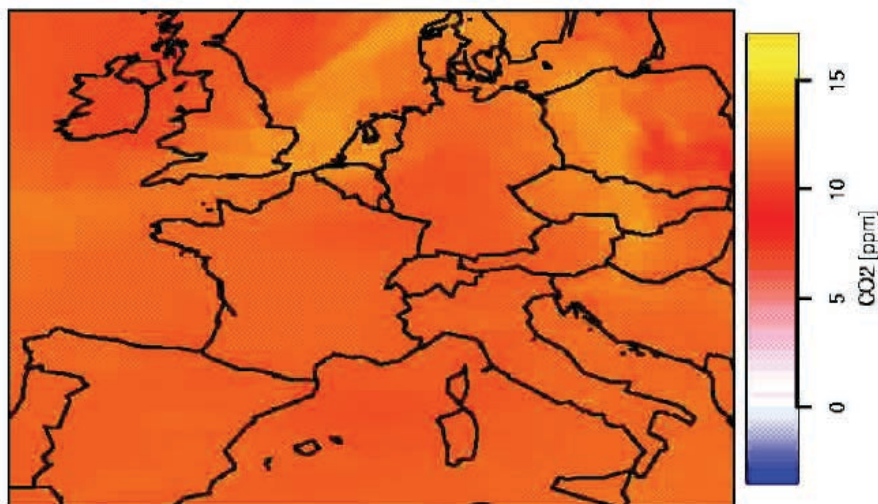
Vegetation-Photosynthesis and Respiration Model, created at MPI-BGC

CO2 at 0.1 km, time 2003-07-02\_00:00:00



WRF+CASA+VPRM, created at MPI-BGC

column average CO2, time 2003-07-02\_00:00:00



WRF+CASA+VPRM, created at MPI-BGC

# Take home messages

- The need to constrain greenhouse gas budgets inevitably leads to the need for the solution of inverse problems
- These inverse problems:
  - Require (intelligently) choosing among many uncomfortable assumptions
  - Are becoming increasingly statistically sophisticated and computationally demanding
  - Done carefully, can lead to fundamental insights with management and policy implications

# Overall inverse problem

*All vary in space  
and time*

$$y = h(z) + \varepsilon_y + \varepsilon_h + \varepsilon_{rep} + \varepsilon_{agg}$$

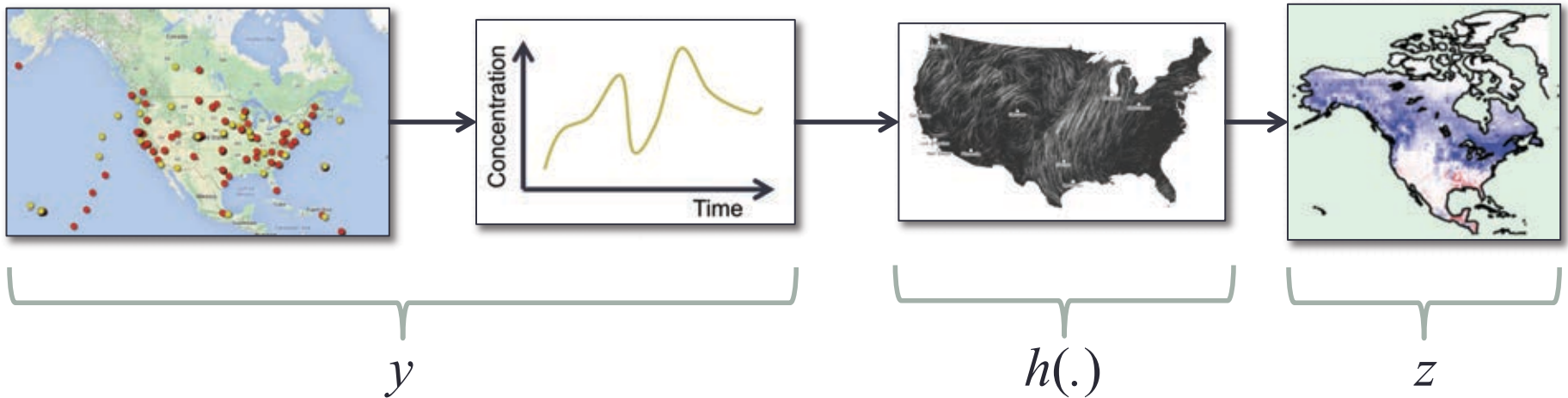
- Find  $z$  given  $y$ , where:
  - $y$ : atmospheric concentration observations (some places, some times)
  - $z$ : surface fluxes (everywhere, all the time)
  - $h(\cdot)$ : atmospheric transport
  - $\varepsilon_y$ : measurement error
  - $\varepsilon_h$ : atmospheric transport model error
  - $\varepsilon_{rep}$ : “representation” error (finite resolution in  $y$ )
  - $\varepsilon_{agg}$ : “aggregation” error (finite resolution in  $z$ )



# Overall inverse problem

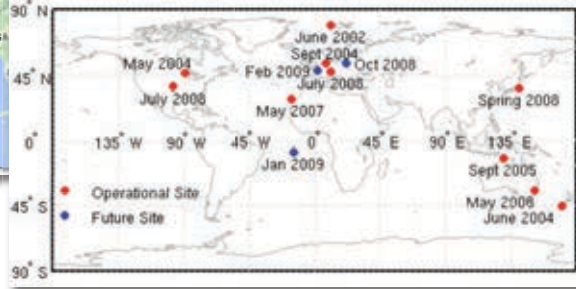
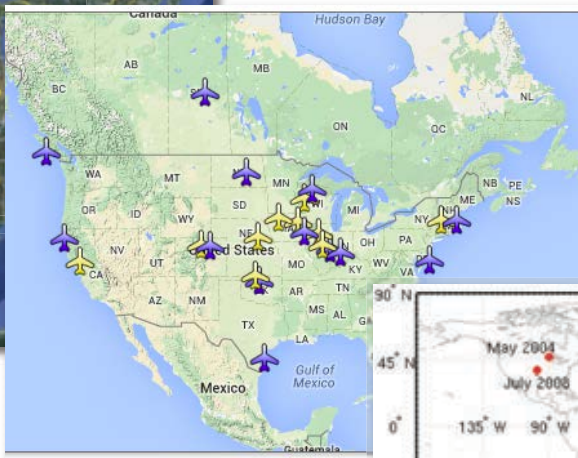
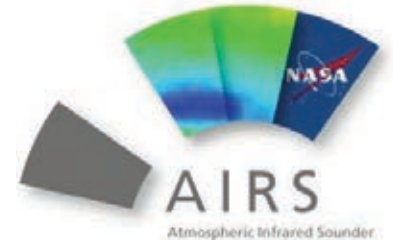
*All vary in space and time*

$$y = h(z) + \varepsilon_y + \varepsilon_h + \varepsilon_{rep} + \varepsilon_{agg}$$



← Causation

→ Inference



Observations, *y*

# Atmospheric transport, $h(\cdot)$

15km ARW WRF, NAM-init -- NCAR/MMM

Fcst: 18 h

Horizontal wind speed

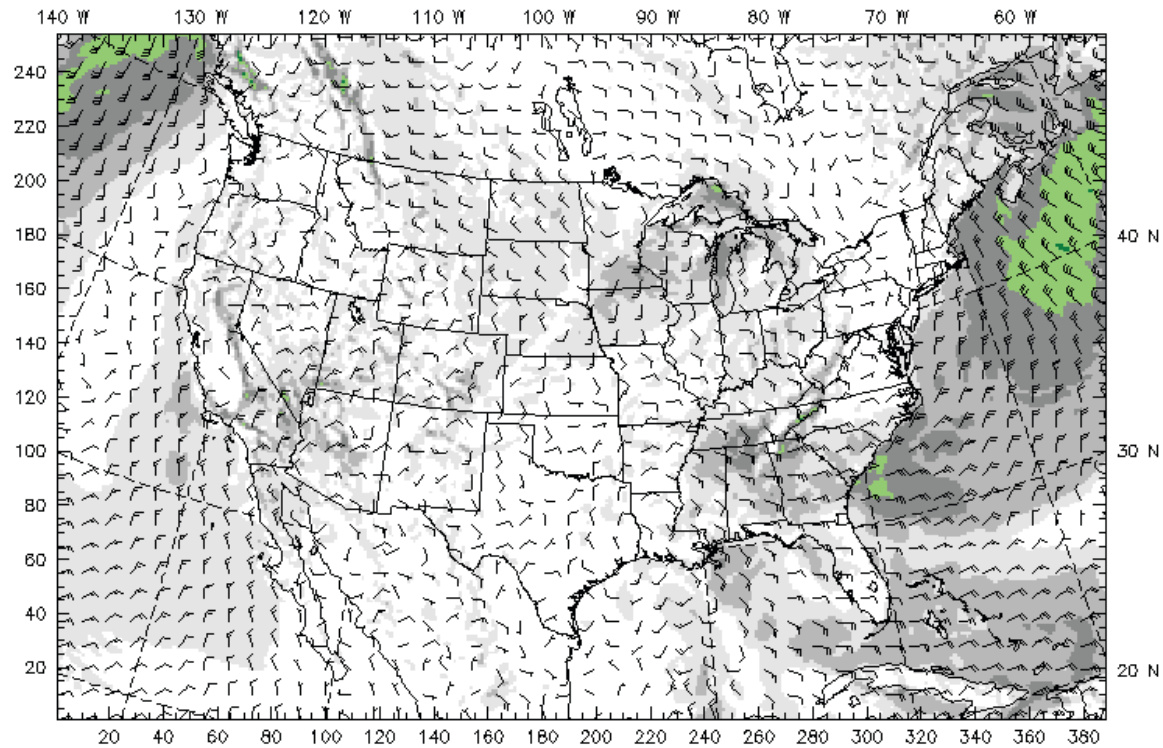
Horizontal wind vectors

Valid: 06 UTC Fri 13 Mar 15 (00 MDT Fri 13 Mar 15)  
at k-index = 39  
sm= 1

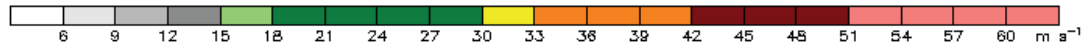
Init: 12 UTC Thu 12 Mar 15

sm= 1

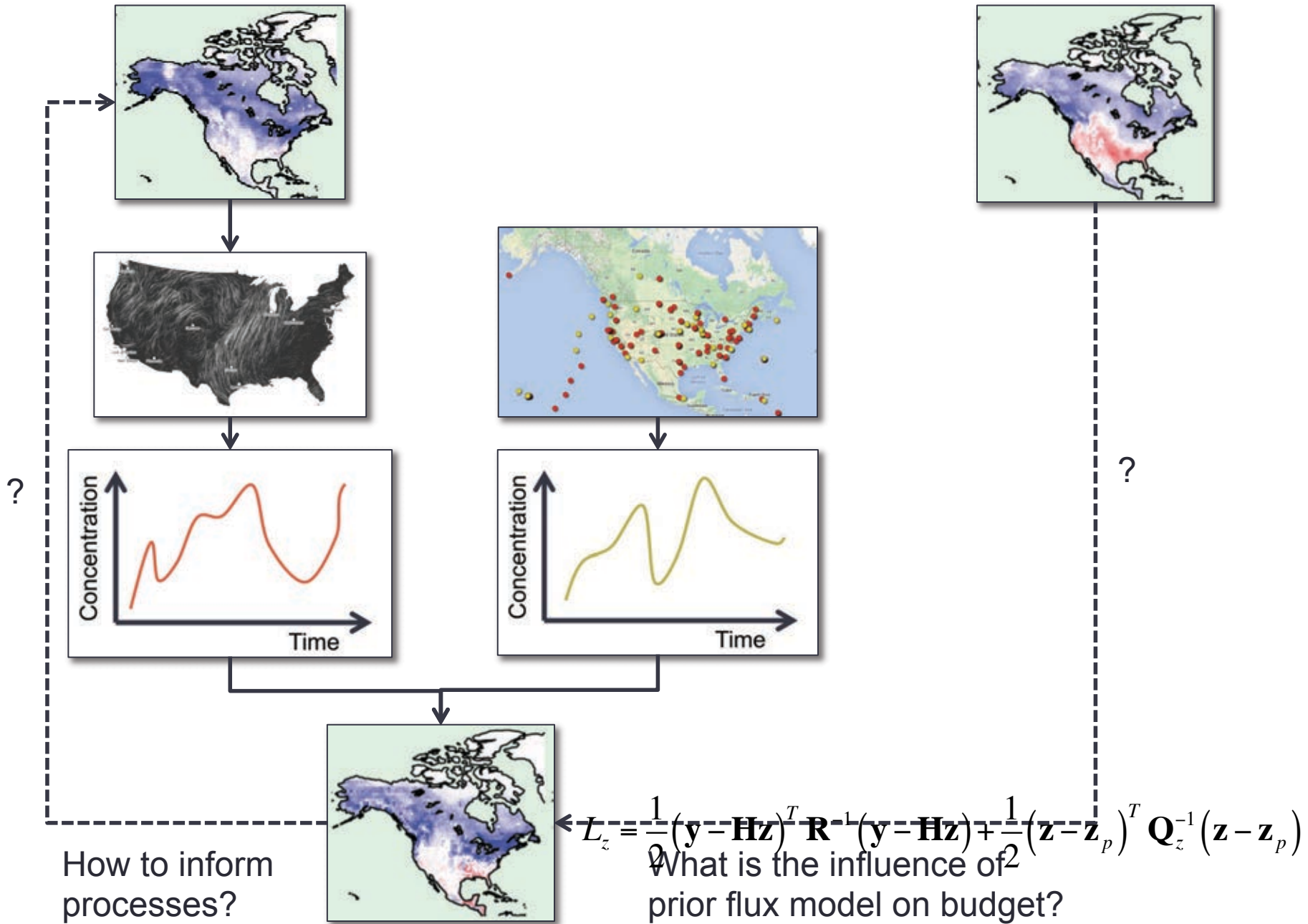
sm= 1



BARB VECTORS: FULL BARB = 10 kts



Model Info: V3.6.1 CU: G-F Ens MP: WDM 6class PBL: YSU SF: Noah LSM 15 km 39 levels 90 sec  
LW: RRTMG SW: RRTMG DIFF: full KM: 2D Smagor DAMP: Rayleigh3 SFLAY: M-O



# Mixed linear model

*All vary in space and time*

$$y = h(z) + \varepsilon_y + \varepsilon_h + \varepsilon_{rep} + \varepsilon_{agg}$$

$$y = \mathbf{H}z + \varepsilon$$

Linear forward model

High spatiotemporal resolution for  $z$

$$z = \mathbf{X}\beta + \xi$$

$$y = \mathbf{H}\mathbf{X}\beta + \mathbf{H}\xi + \varepsilon$$

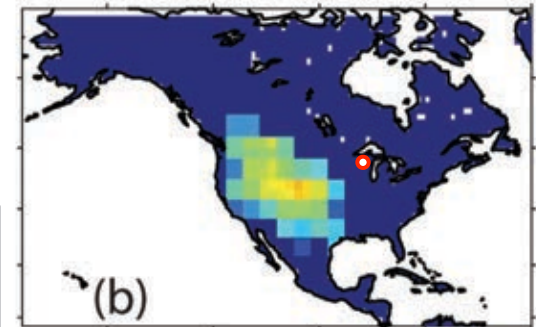
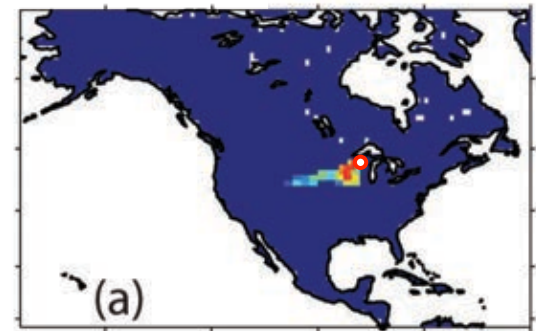
BIC for model selection  
(space-time correlated residuals)

$$\xi \sim N(0, \mathbf{Q})$$

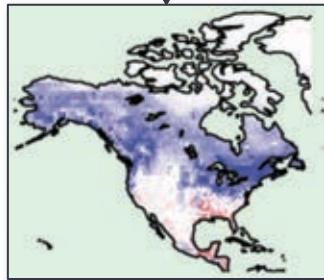
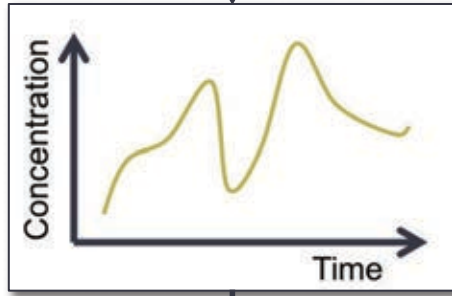
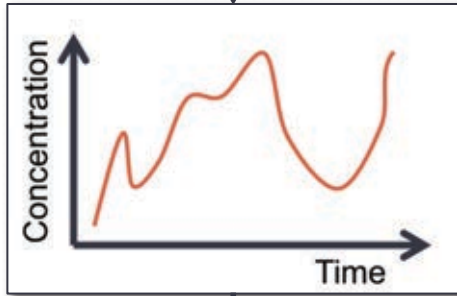
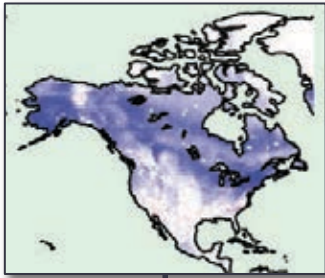
Stationary in space, nonstationary in time, parametric model, not sparse

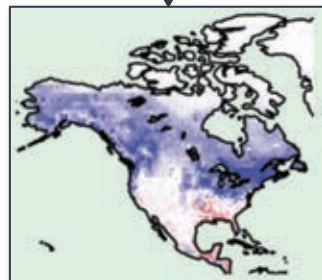
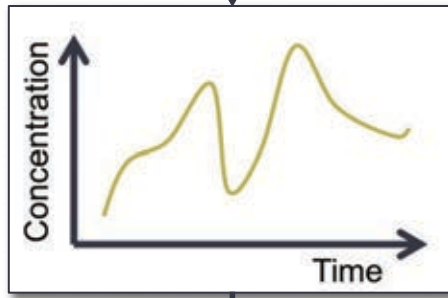
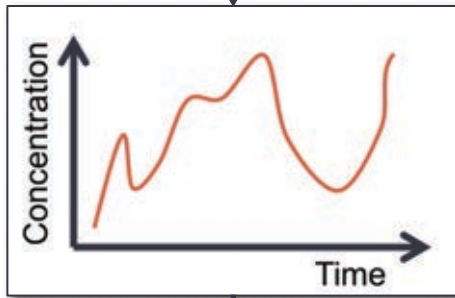
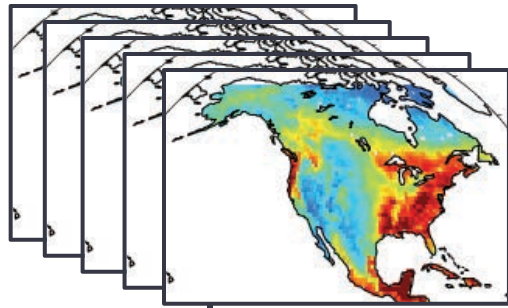
$$\varepsilon \sim N(0, \mathbf{R})$$

Independent, variable variance



ReML for parameter estimation





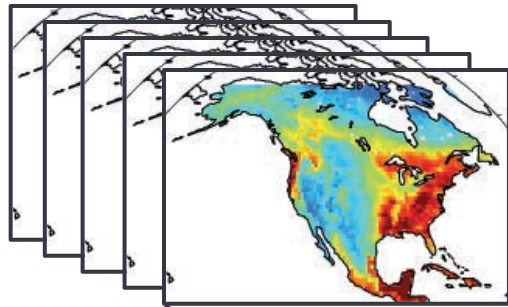
**Bigger, better, faster, more!**  
Michalak et al. (2004), Gourdj  
et al. (2010), Chatterjee  
(2012), Yadav (2013a,b),  
Miller (2014a)

**Biospheric CO<sub>2</sub> budgets:**  
Mueller et al. (2008),  
Gourdji et al. (2008, 2012)

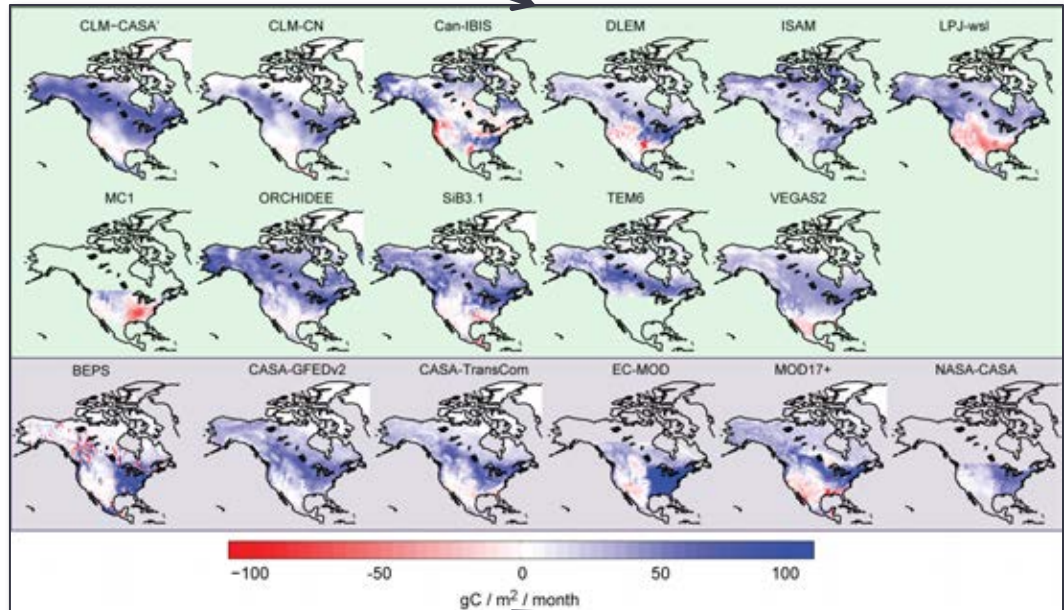
**Anthropogenic CO<sub>2</sub> budgets:**  
Shiga et al. (2014),  
Yadav et al. (in revision)

**CH<sub>4</sub> and N<sub>2</sub>O budgets:**  
Miller et al. (2012, 2013,  
2014b)

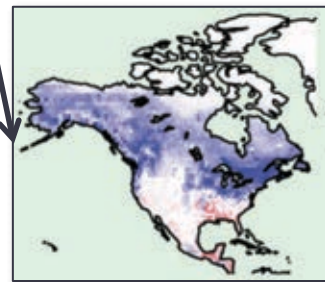
Can provide more  
objective budget  
estimates



Can evaluate  
models' process  
representations



Can provide  
process  
information  
directly at  
target scales



Can confront  
models with  
independent flux  
estimate



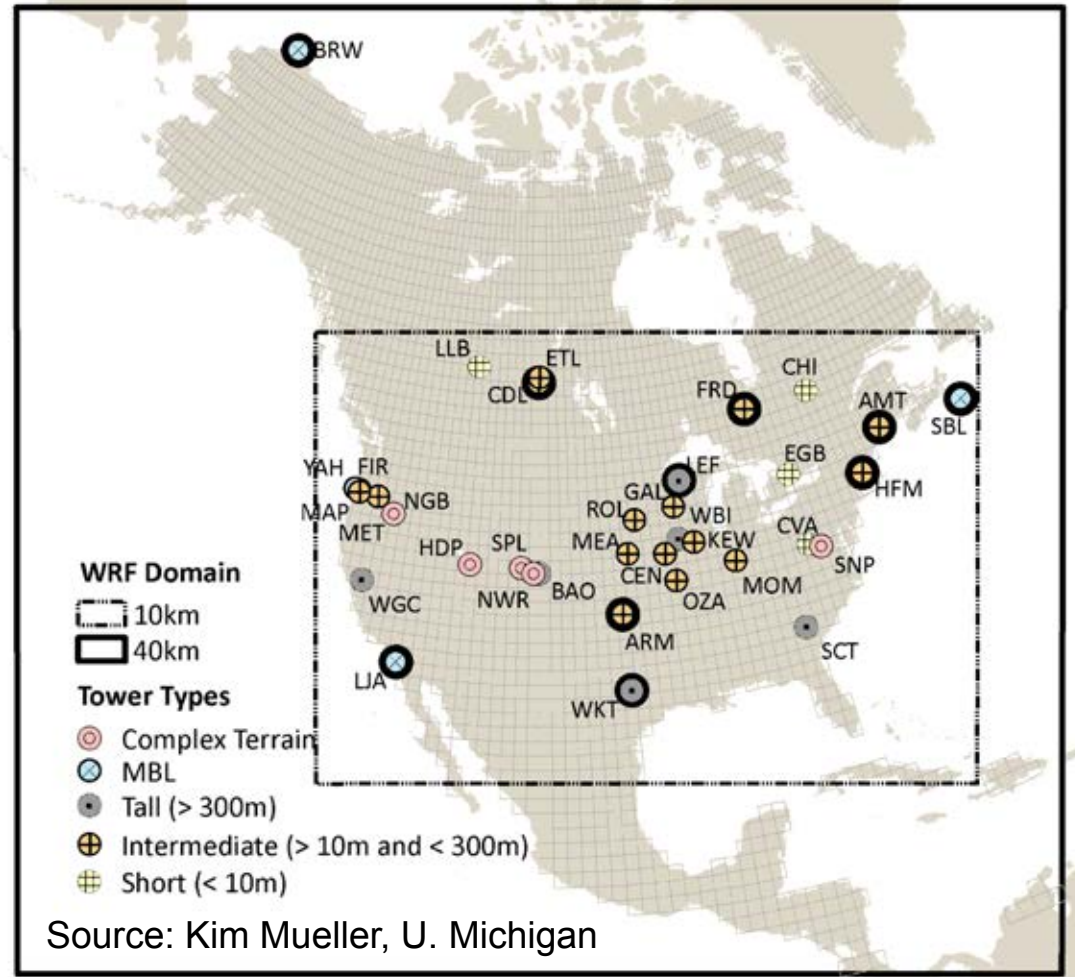
# Increasing cost of inversions

Regional CO<sub>2</sub> inversions over North America for one year at 1° x 1°; 3-hourly

y:  $\sim 10^5$

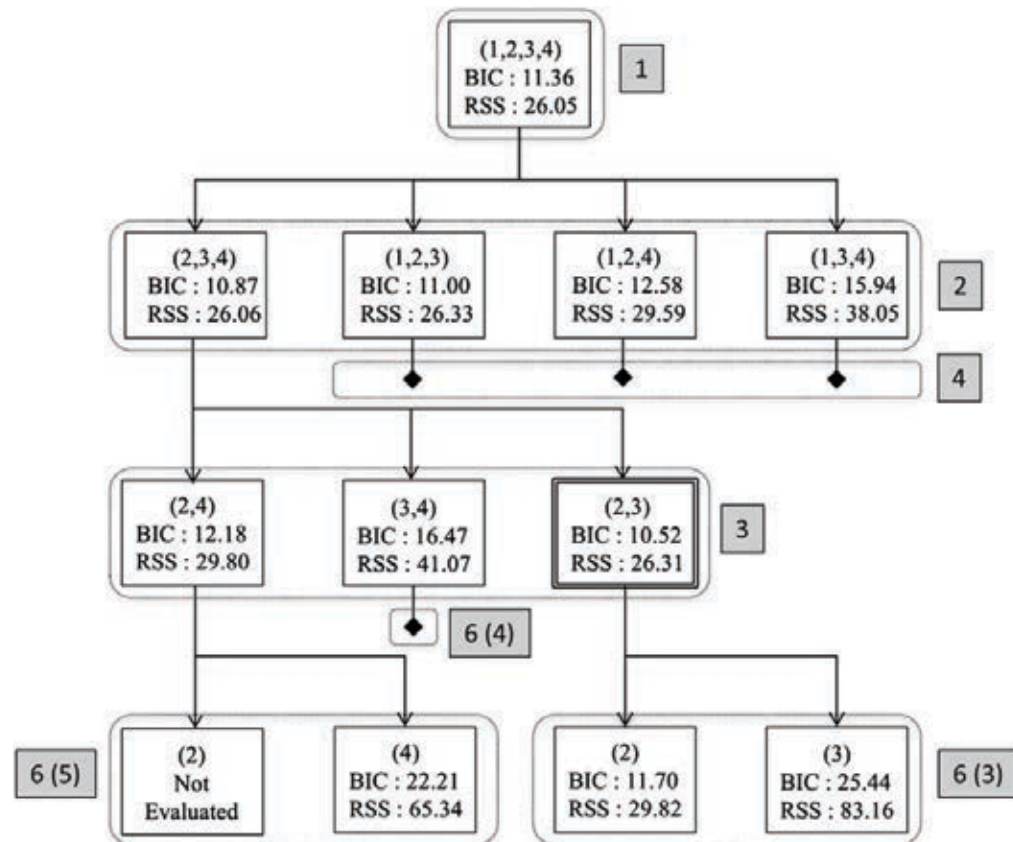
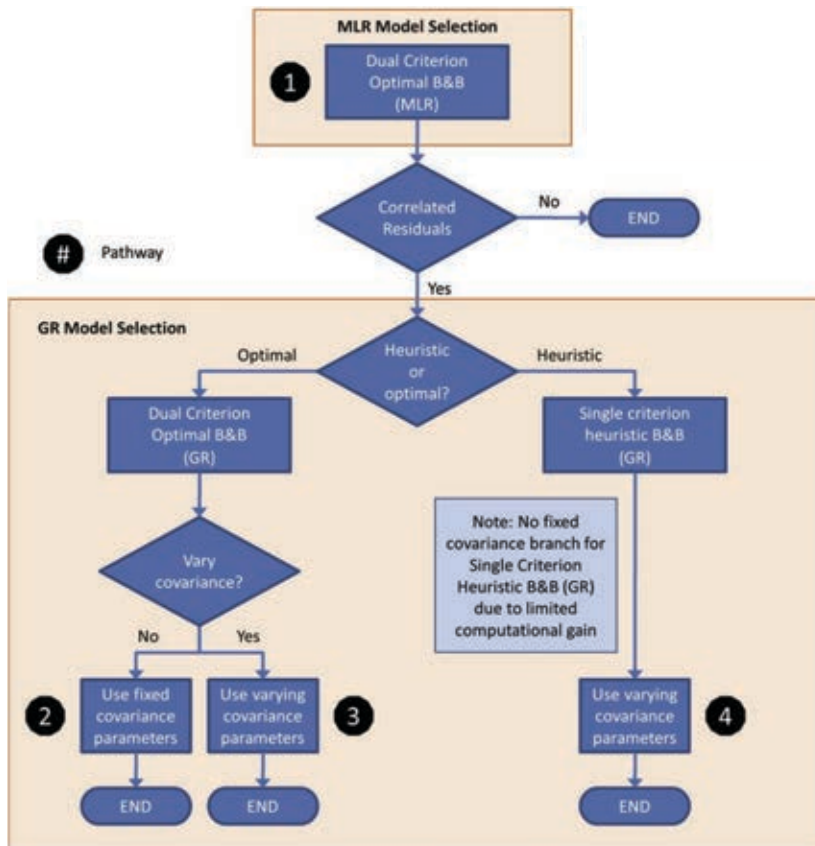
z:  $\sim 10^6$

X:  $\sim 10^2$



(H:  $\sim 10^5 \times 10^6$ ; Q:  $\sim 10^6 \times 10^6$ )

# Branch & bound algorithm for model selection



$k$  covariate yields  $2^k$  candidate models

# Matrix multiplication & posterior covariances

$$\mathbf{y} \sim N(\mathbf{H}\mathbf{X}\beta, \mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R})$$

$$\hat{\mathbf{z}} \sim N(\Lambda\mathbf{y}, (\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} + \mathbf{Q}^{-1})^{-1})$$

$$\mathbf{Q} = \sigma_s^2 \left[ \overbrace{\exp\left(-\frac{\mathbf{X}_\tau}{l_\tau}\right)}^{\text{temporal covariance } (\mathbf{D})} \right] \otimes \left[ \overbrace{\exp\left(-\frac{\mathbf{X}_s}{l_s}\right)}^{\text{spatial covariance } (\mathbf{E})} \right],$$

$$\mathbf{H}_{(n \times m_\tau m_s)} = \left( \underbrace{\mathbf{h}_1}_{(n \times m_s)} \quad \underbrace{\mathbf{h}_2}_{(n \times m_s)} \quad \dots \quad \underbrace{\mathbf{h}_{m_\tau}}_{(n \times m_s)} \right)$$

$$\mathbf{H}\mathbf{Q}_{(n \times m_\tau m_s)} = \left( \underbrace{\left( \sum_{i=1}^{m_\tau} \mathbf{h}_i d_{(i,1)} \right) \mathbf{E}}_{(n \times m_s)} \quad \underbrace{\left( \sum_{i=1}^{m_\tau} \mathbf{h}_i d_{(i,2)} \right) \mathbf{E}}_{(n \times m_s)} \quad \dots \quad \underbrace{\left( \sum_{i=1}^{m_\tau} \mathbf{h}_i d_{(i,m_\tau)} \right) \mathbf{E}}_{(n \times m_s)} \right)$$

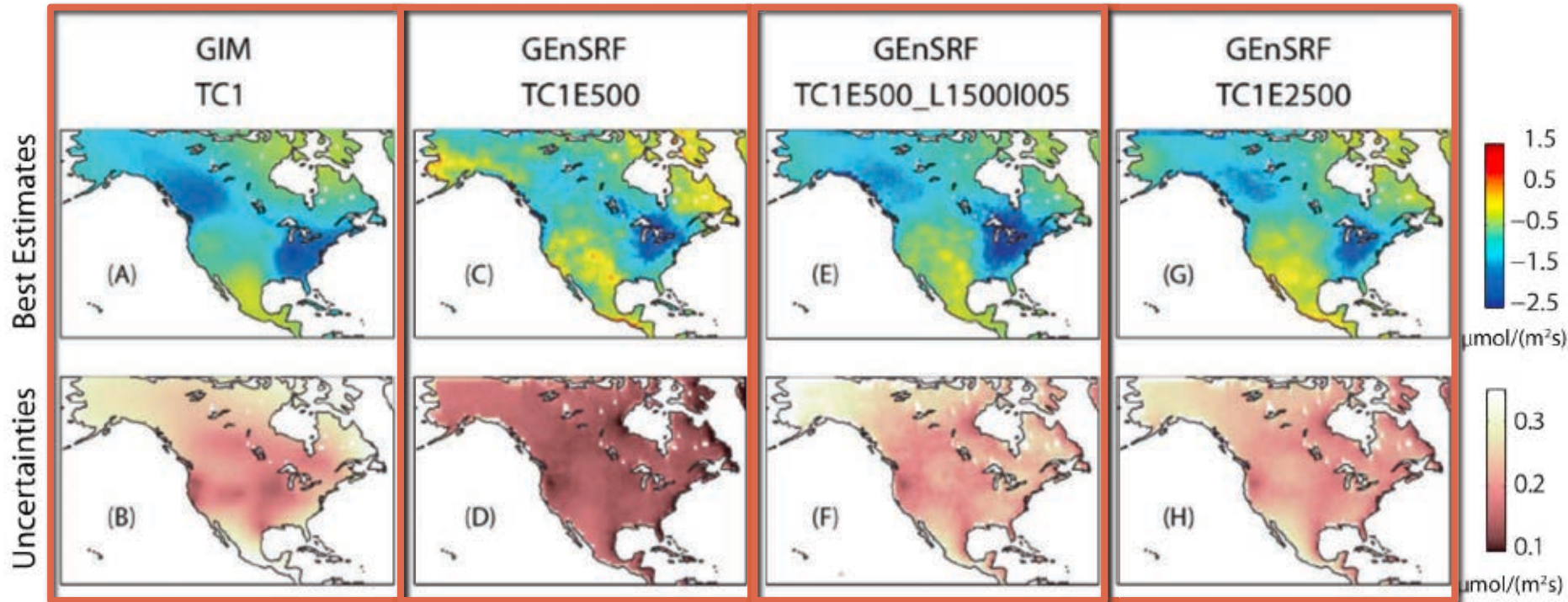
$$\mathbf{Q}_{\text{sum}}(m_s \times m_\tau) = \left( \left( \sum_{j=t_l}^{t_u} \sum_{i=t_l}^{t_u} d_{(i,j)} \right) \mathbf{E} \right),$$

$$(\mathbf{H}\mathbf{Q})_{\text{sum}} = \left( \sum_{j=t_l}^{t_u} \left( \sum_{i=1}^{m_\tau} \mathbf{h}_i d_{(i,j)} \right) \mathbf{E} \right)_{(n \times m_s)},$$

$$\bar{\mathbf{V}}_{\hat{\mathbf{s}}} = \frac{(\mathbf{Q}_{\text{sum}} - (\mathbf{H}\mathbf{Q})_{\text{sum}}^T (\mathbf{H}\mathbf{Q}\mathbf{H}^T + \mathbf{R})^{-1} (\mathbf{H}\mathbf{Q})_{\text{sum}})}{k^2},$$

Both algorithms require  $O(n^{2.5})$  operations instead of  $O(n^3)$  for direct solution.

# Ensemble SRF approaches

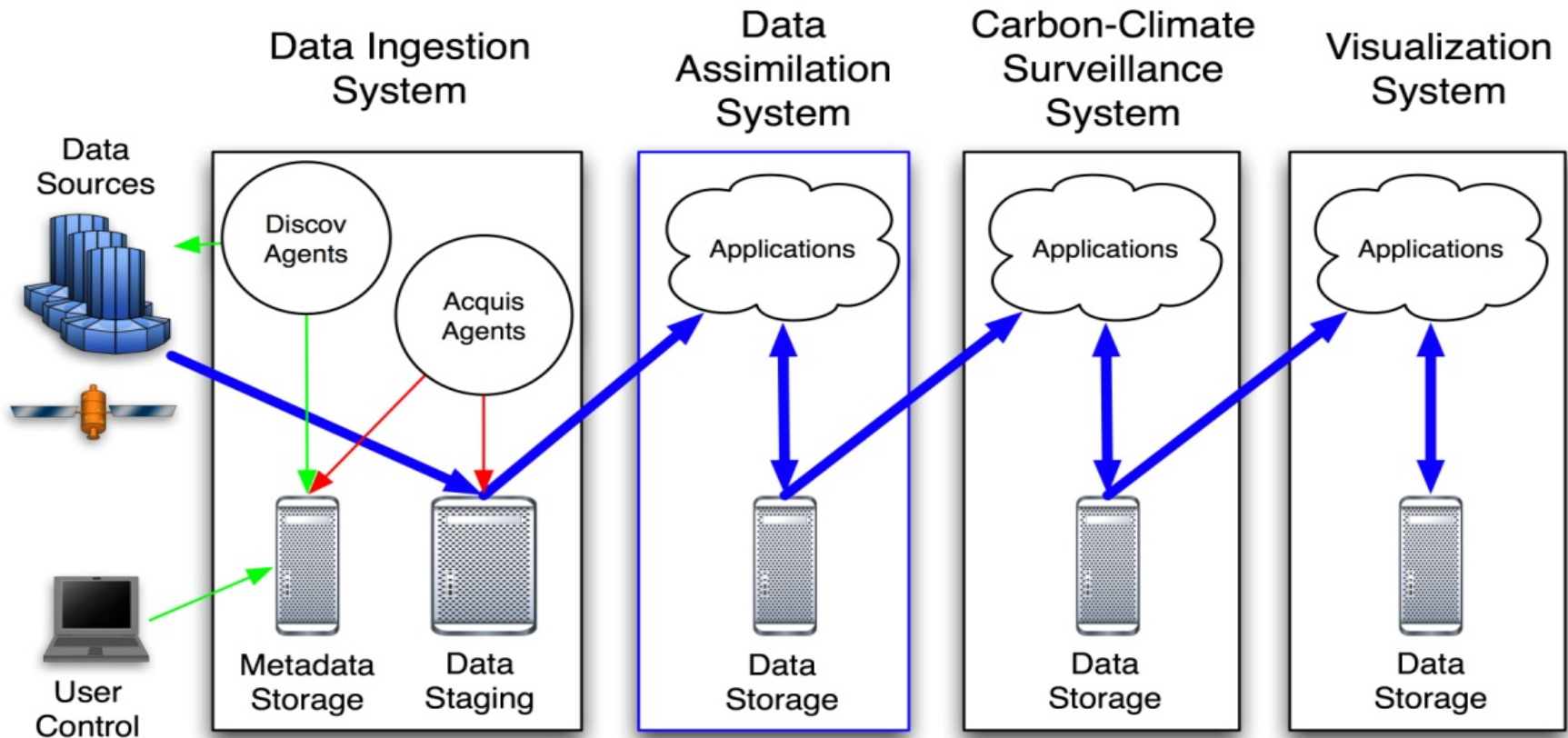


**Figure 4.** TC1 (top) flux estimates and (bottom) associated uncertainties aggregated to the monthly scale for (a and b) GIM and (c–h) three different GEnSRF runs.

## Features:

- No dynamical model
- Kalman smoother
- Heterogeneous (in space and time) observational network

# Real-Time Large-Scale Parallel Intelligent CO<sub>2</sub> Data Assimilation System

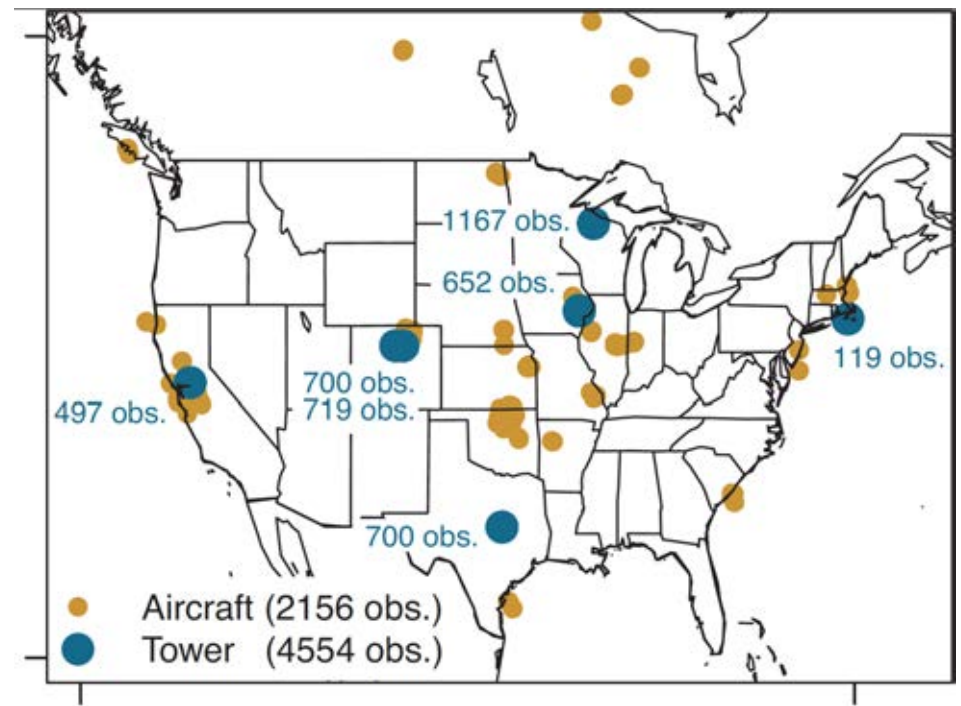
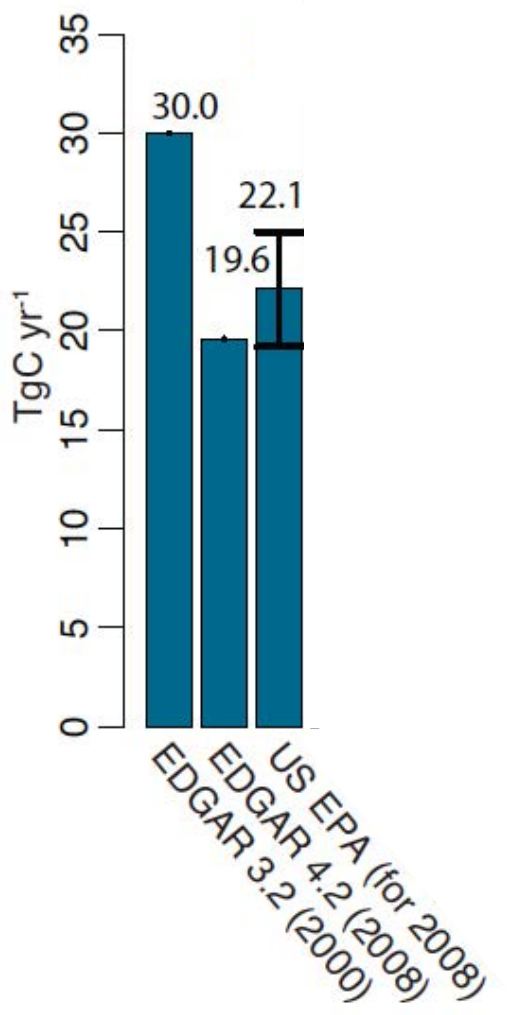


# Take home messages

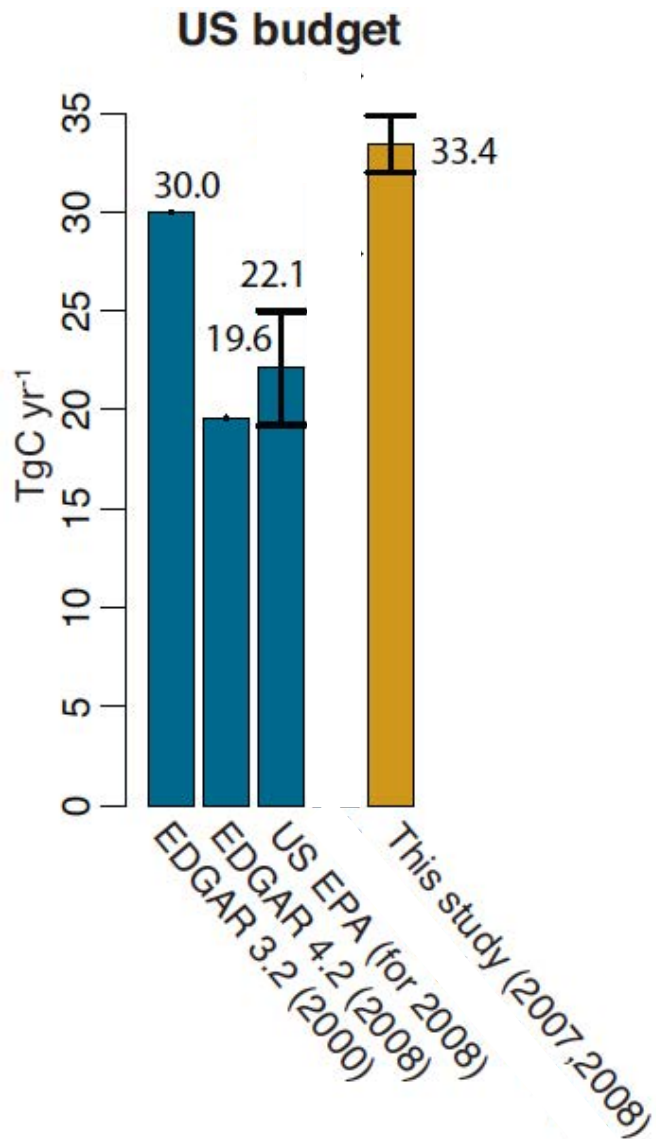
- The need to constrain greenhouse gas budgets inevitably leads to the need for the solution of inverse problems
- These inverse problems:
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# U.S. anthropogenic methane emissions

### US budget

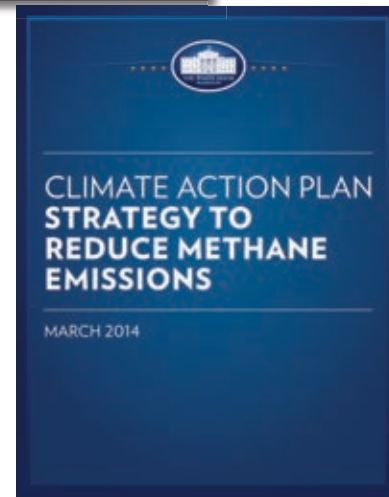


# U.S. methane emissions



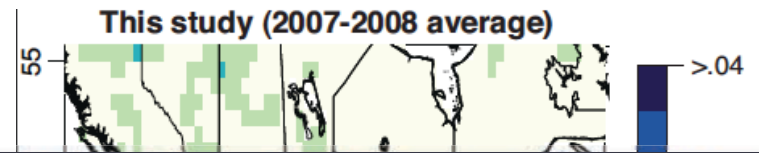
U.S. anthropogenic methane emissions are **50%** higher than EPA estimates

Methane emissions in TX / OK / KS are **triple** of what inventories suggest, and a **quarter** of total U.S. emissions





# Estimated methane fluxes



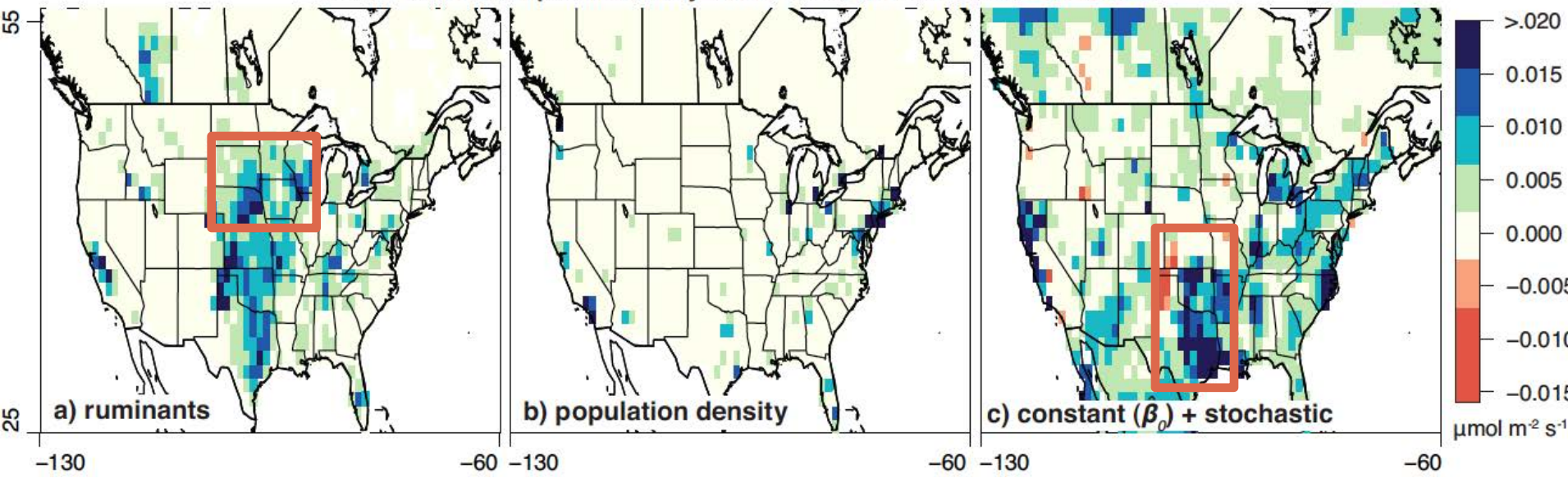
Ruminant source is nearly *double* what inventories suggest. Oil and gas emissions are *5x* those in EDGAR 4.2 for TX/OK/KS.



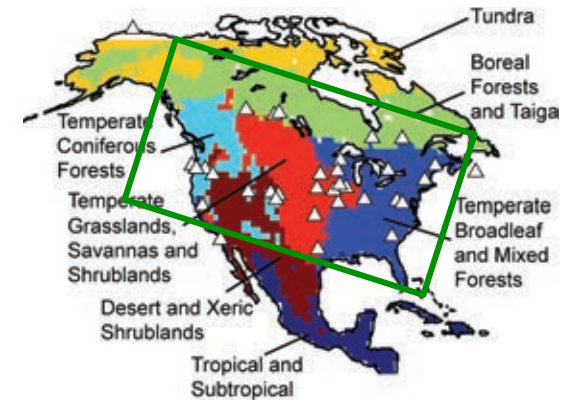
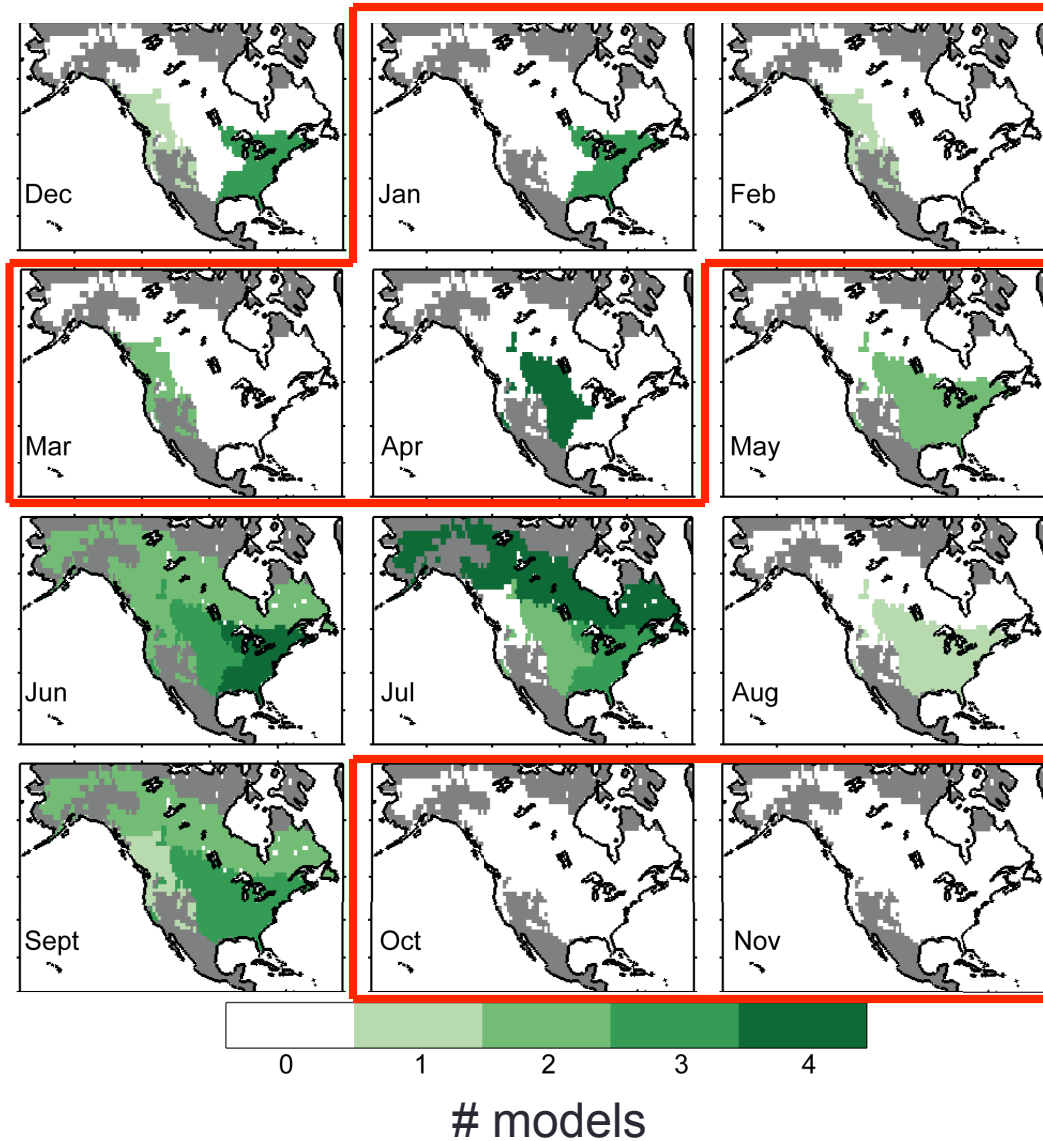
3.4 ± 0.7 TgC yr<sup>-1</sup>

3.7 ± 2.0 TgC yr<sup>-1</sup>

Contribution of spatial activity datasets to the estimated emissions

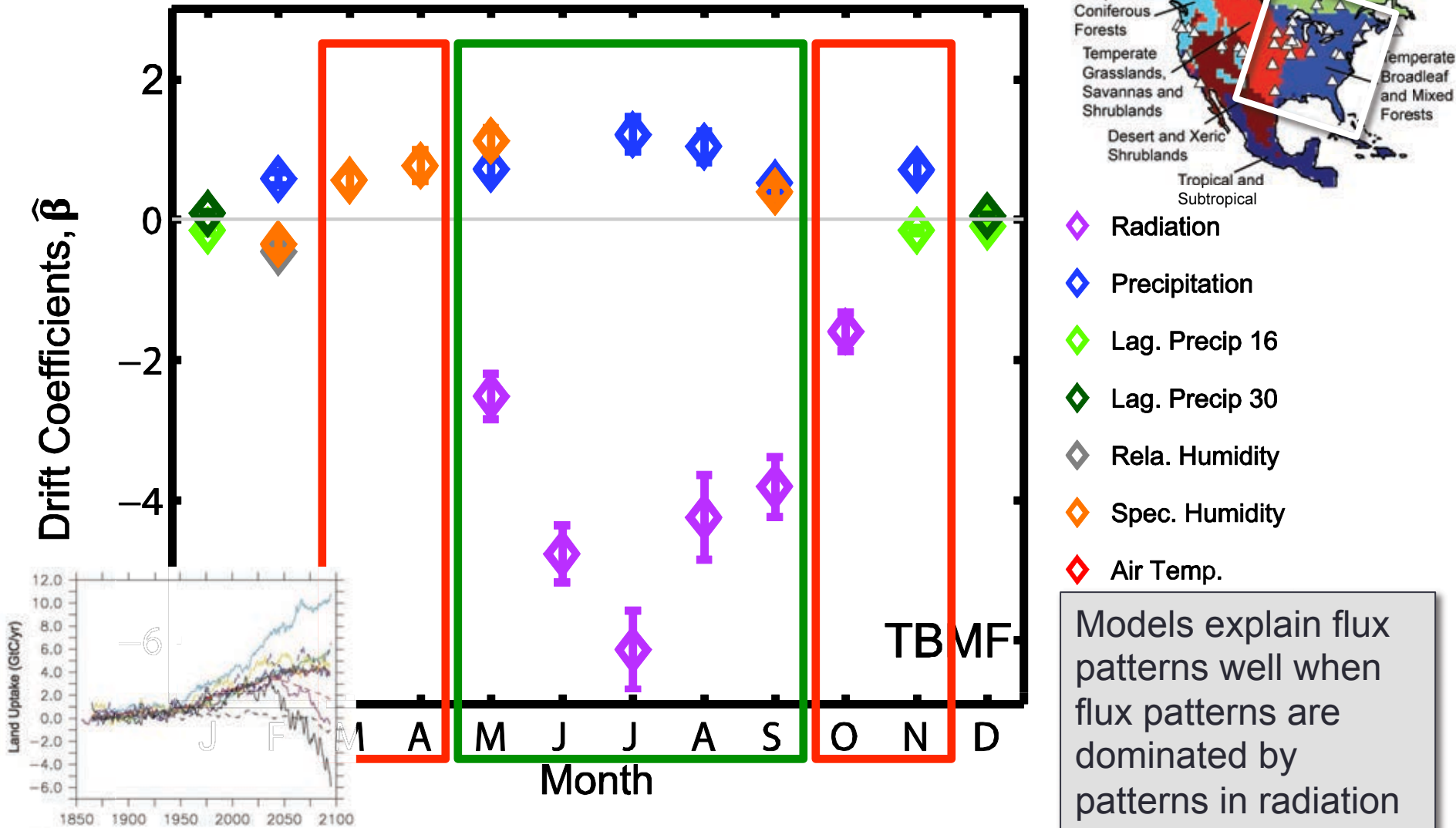


# Confronting model flux patterns with obs



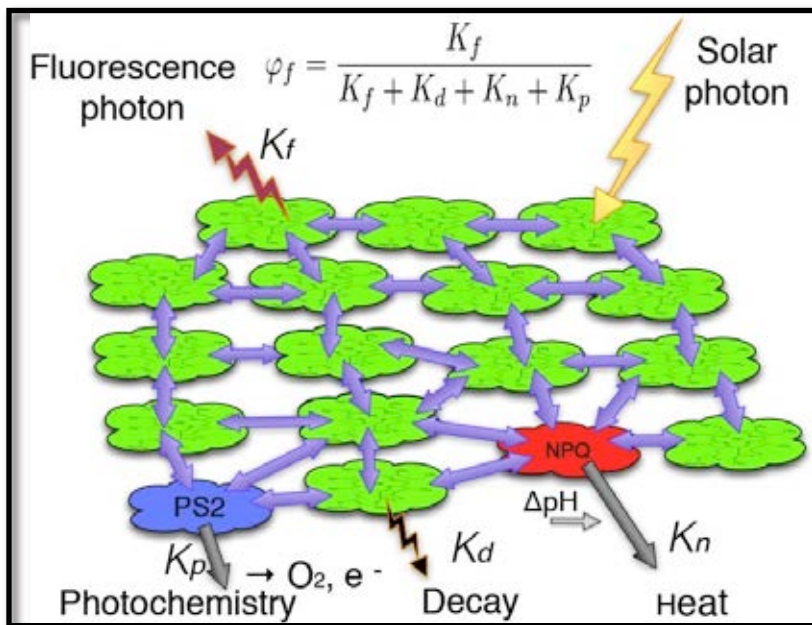
Models' flux patterns do not explain observed variability in atmospheric observations for much of the year, but they do better during growing season.

# Providing process information directly at target scales

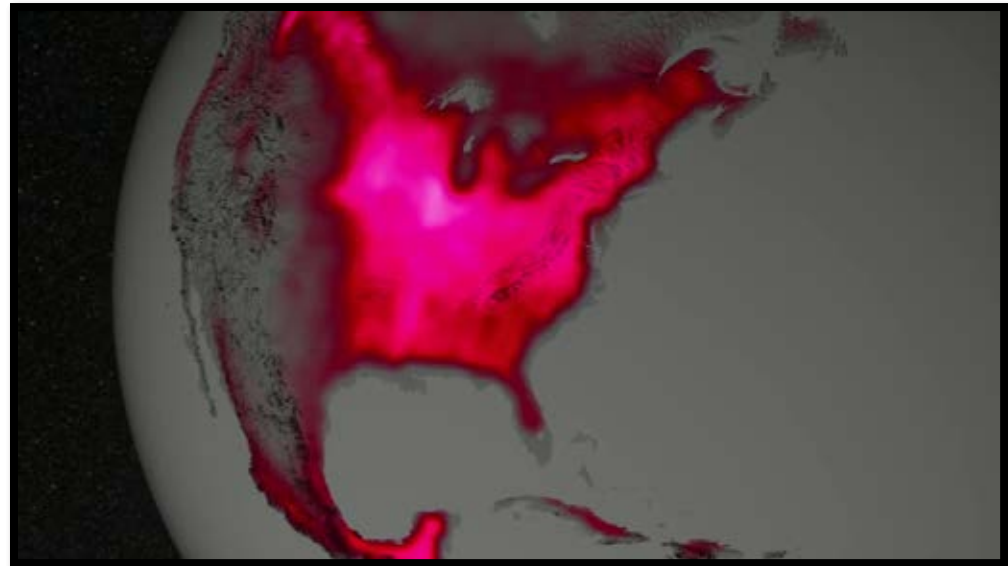


# Solar Induced Fluorescence

SIF emitted during photosynthesis and is therefore potentially a promising measure of GPP

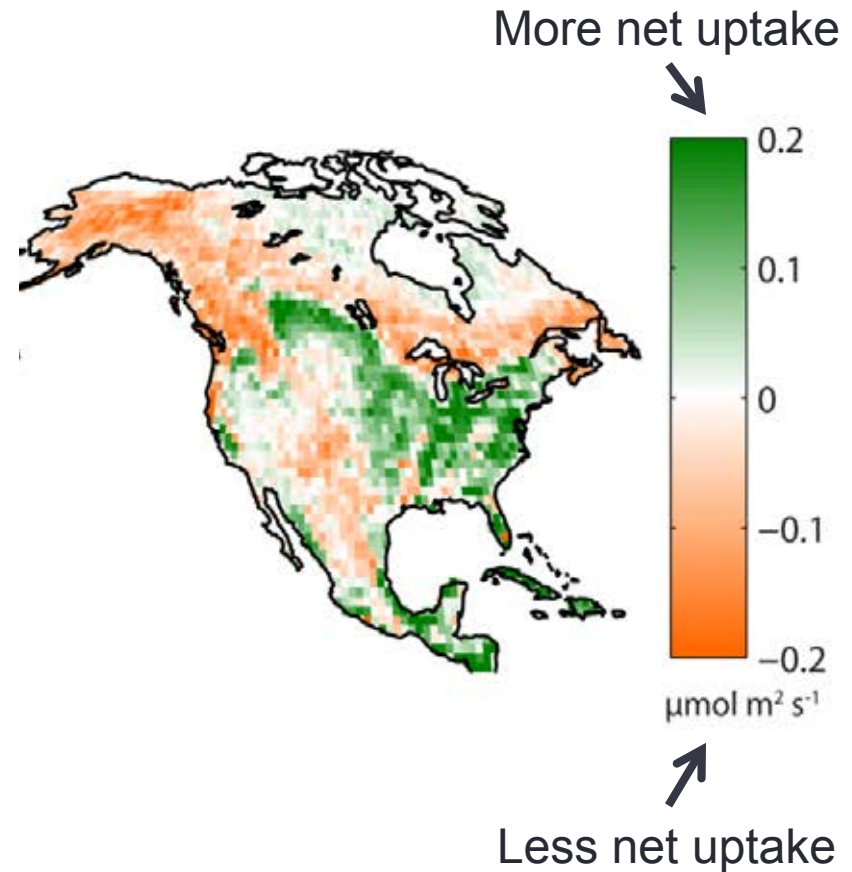


Source: Frankenberg, 2011

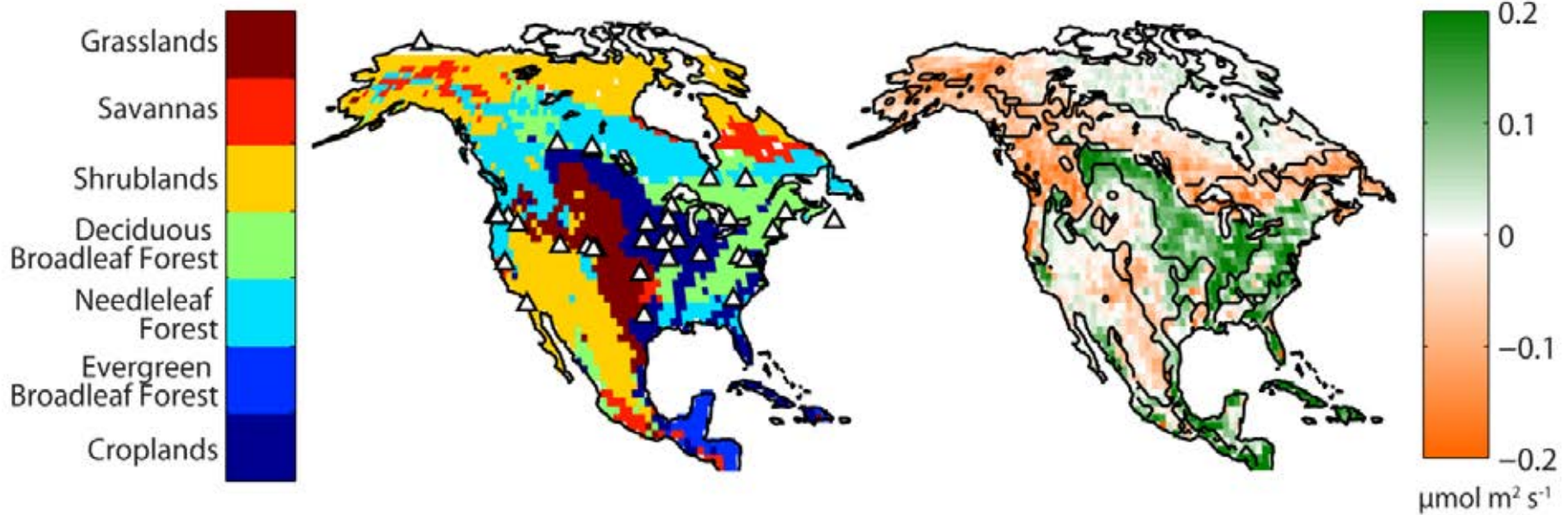


Source: [http://www.nasa.gov/press/goddard/2014/march/satellit-e-shows-high-productivity-from-us-corn-belt/#.U8QK4\\_IdV8G](http://www.nasa.gov/press/goddard/2014/march/satellit-e-shows-high-productivity-from-us-corn-belt/#.U8QK4_IdV8G)

# Differences at $1^\circ \times 1^\circ$ , aggregated over March to October



# Differences at $1^\circ \times 1^\circ$ , aggregated over March to October



Informing inversions with SIF leads to redistribution of carbon sink, with increased sink in croplands and reduced sink in needleleaf forests

# Take home messages

- The need to constrain greenhouse gas budgets inevitably leads to the need for the solution of inverse problems
- These inverse problems:
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  - Done carefully, can lead to fundamental insights with management and policy implications

# Acknowledgments

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